Transport Coefficients of Hadron Matter at Finite Temperature

Andres Ortiz (*REU student from University of Texas at El Paso*) Dr. Ralf Rapp, Dr. Felix Riek
Cyclotron Institute, Texas A&M University

Introduction

Quantum Chromodynamics (QCD) is the theory that describes quarks, gluons and their interactions. The force of attraction between two quarks becomes constant after some distance requiring an infinite amount of energy to separate them. This phenomenon is known as quark confinement and is the reason why we only see quarks in bound states forming hadrons but never free or isolated. However, at high temperature –around 200MeV- QCD predicts that hadron matter will undergo a phase transition into plasma of deconfined quarks and gluons (fig .1).

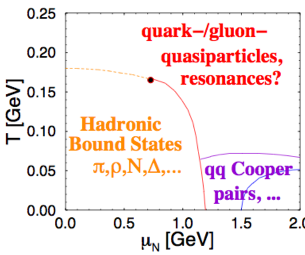


Figure 1.
QCD Phase Diagram

The Super Proton Syncrotron (SPS) at CERN and Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory are trying to recreate a quark gluon plasma by colliding accelerated gold nuclei at very high energies. The plasma is expected to exist for a very short period of time, about 10^{-22}s. One way of measuring properties of the QGP is by means of the dilepton pairs produced during its existence. These are created when a quark and an anti-quark of the medium annihilate producing a photon, which then separates into a dilepton pair (fig. 2). Since leptons are not subject to the strong force they interact very weakly with the plasma making them an excellent probe for studying the QGP.

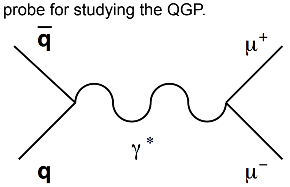


Figure 2.
Dilepton pair production from quark anti-quark anihilation

The dilepton production rate is related to the electrical conductivity by the imaginary part of the current-current correlator. According to classical kinetic theory, the electrical conductivity is related to the shear viscosity by a simple formula. Although electrical conductivity is not easy to measure viscosity is indirectly accessible via thermodynamic simulations and so providing a good test for theoretical models.

Electrical Conductivity and Shear Viscosity

The electrical conductivity is a transport coefficient that describes the response of a system to an electric field. As mentioned before the dilepton production rate is related to the electrical conductivity.

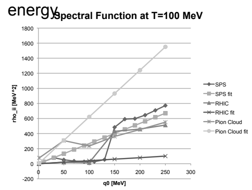
$$\frac{dN}{d^4x d^4q} = -\frac{\alpha_{em}^2}{\pi^3 M^2} f^B(q_0;T)\frac{1}{3}\mathrm{Im}\Pi_{em}(M,q;\mu_B,T)$$

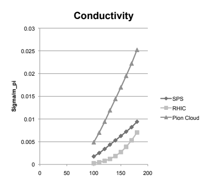
The latter can be obtained from the spectral function at low energies by means of the Kubo formula:

$$\sigma = \frac{e^2}{6}\lim_{q_0\to 0}\frac{\rho_{ii}(q_0,0)}{q_0} \qquad \rho_{ii} = -\mathrm{Im}\Pi_{em}$$

here rho is the spectral function.

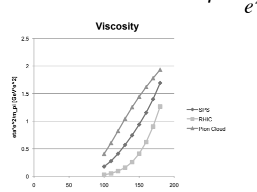
Theoretical calculations (1) of the spectral function at SPS and RHIC conditions are extrapolated to obtain the slopes at zero frequency. The slope is obtained from a linear fit to the first points considering the fact that the spectral function goes to zero at zero energy.

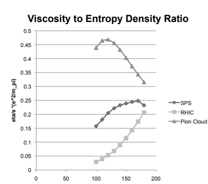




A relationship between electrical conductivity and shear viscosity is established from classical kinetic theory. Hadronic gas can be treated as being made of non-interacting constituents if we consider resonances as components of the gas themselves. This justifies the use of kinetic theory.

$$\eta = \frac{k_B m}{e^2}\sigma T$$





Euclidean Correlators

Euclidean correlators can be calculated from the spectral functions:

$$\Pi_\alpha(\tau,q) = \int_0^\infty \frac{dq_0}{\pi}\rho(q_0,q)\frac{\cosh(q_0(\tau-1/2T))}{\sinh(q_0/2T)}$$

The low energy region of the spectral function is the same one used for the calculations of electrical conductivity. The high energy part is given by:

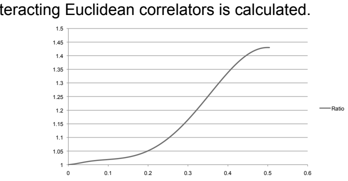
$$\rho_{cont}(q_0) = \frac{q_0^2}{8\pi}\frac{1}{1+\exp[(E-q_0)/\delta]}\left(1+\frac{0.22}{\ln[1+q_0/(0.2GeV)]}\right) \quad (2)$$

where E=1.3GeV and δ=0.2GeV.

Also, the Euclidean correlator for the non-interacting case is calculated. In this case the spectral function is given by:

$$\rho(q_0) = \frac{q_0^2}{8\pi}$$

Integrals are solved numerically by summing areas of small rectangles. The spacing between rectangles is determined from the maximum value of the integrand. Finally, the ratio of in-medium to non-interacting Euclidean correlators is calculated.



Conclusions and Further Work

The results for the Euclidean correlator ratio show surprising agreement with very recent lattice calculations (3), which is evidence that the model used throughout this work is correct. However, there is still some uncertainty on the calculation of transport coefficients due to systematic errors.
A reasonable ansatz for the spectral function at low energies will be used to extract conductivity involving less uncertainty.

Acknowledgements

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References

1. R. Rapp and J. Wambach, Eur. Phys. J. A6 (1999) 415
2. R. Rapp Eur. Phys. J. A18 (2003) 459-462
3. F. Karsch Talk at Lattice 2010 Conference

Cyclotron Institute
Texas A&M University